

# Content of this Chapter

- **Fundamentals of Epitaxial Growth**
- **Self-Organized Growth of Semiconductor Quantum Dots**
- **Thin Film Growth on Flat or Vicinal Substrates**
- **Doping Engineering of Semiconductors**

# Thin Film Growth on Flat or Vicinal Substrates

- **Kinetic Instabilities**
- **Homoepitaxy vs. Heteroepitaxy**
- **Critical Thicknesses in Heteroepitaxy on Flat Substrates**
- **Step Bunching and Step Suppression on Vicinal Substrates**

# Current Understanding of Thin Film Growth

- ◆ Thin film growth is a nonequilibrium process.
- ◆ A specific growth mode is an interplay between thermodynamics and growth kinetics.

**Thermodynamics** (free energy minimization)



**Growth kinetics** (various atomic rate processes)

$$r_i = v_0 \exp\{-V_i/k_B T\}.$$

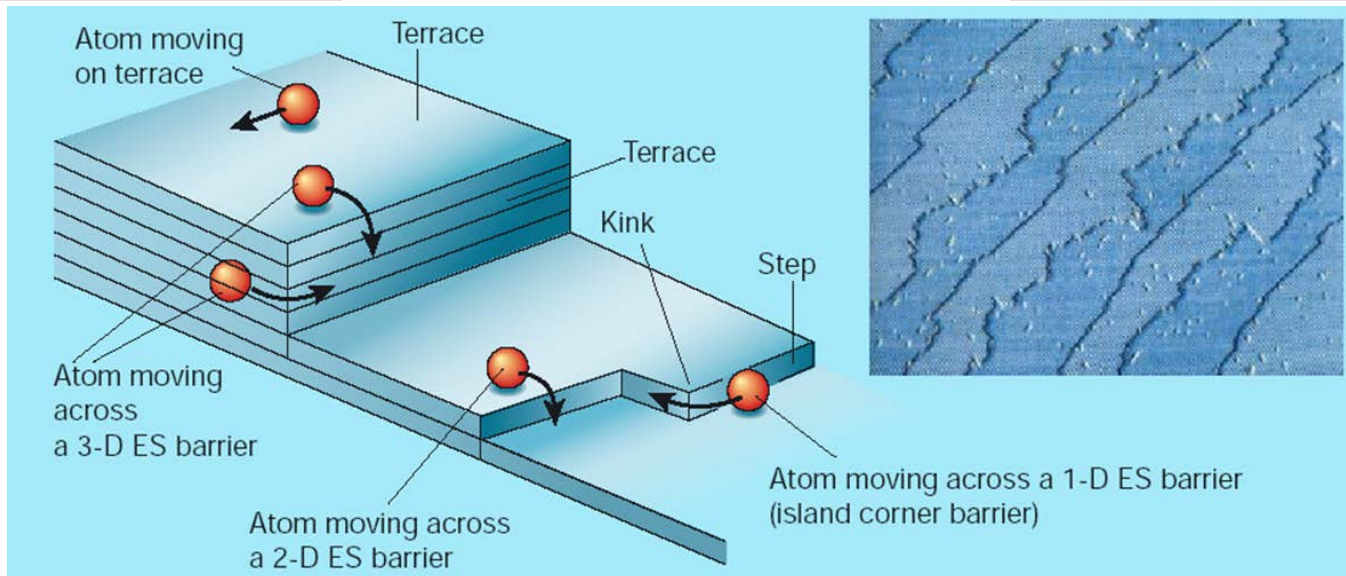
# Philosophy

If we can establish **EVERY** correspondence between

**Specific Atomic  
Rate Process**

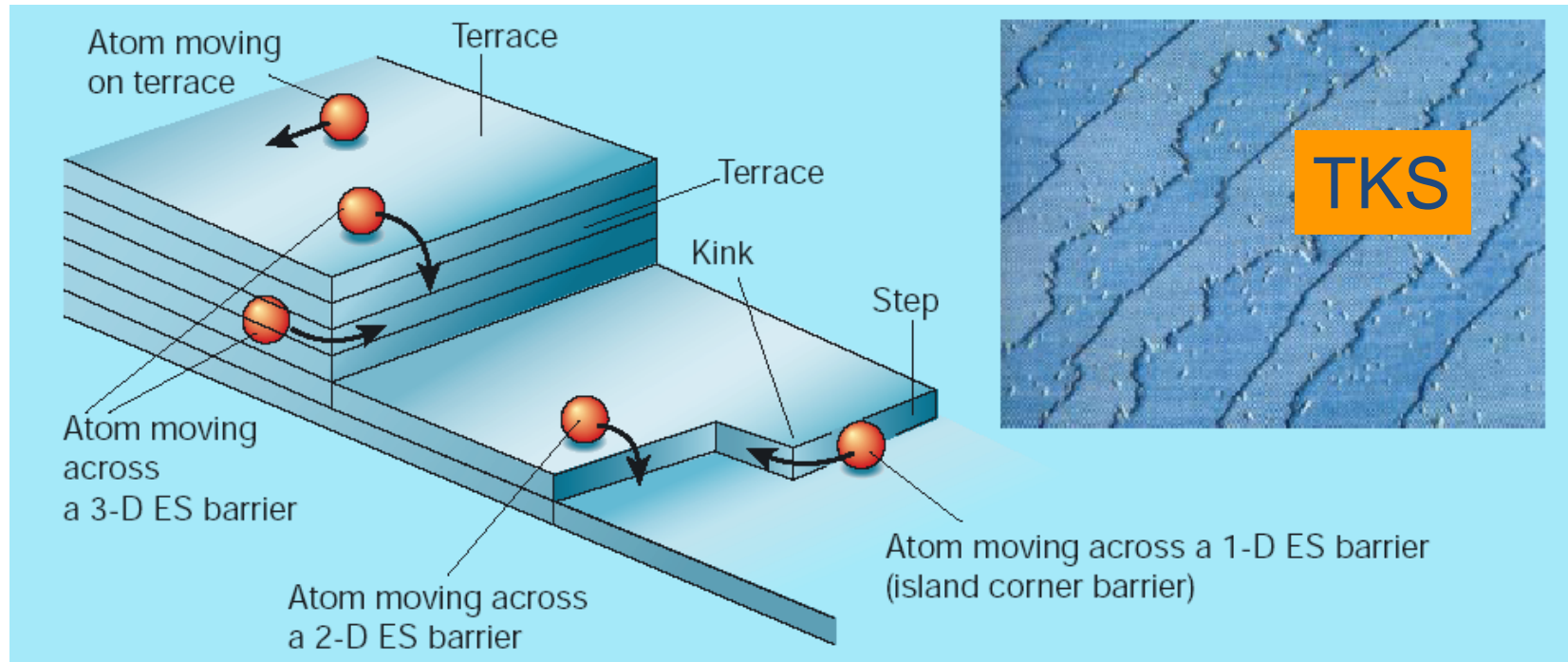


**Morphological  
Evolution**



then we should be able to select a preferred growth mode via precise control of the profile with various rate processes.

# Multiscale Description of Crystal Growth: Important Atomic Rate Processes

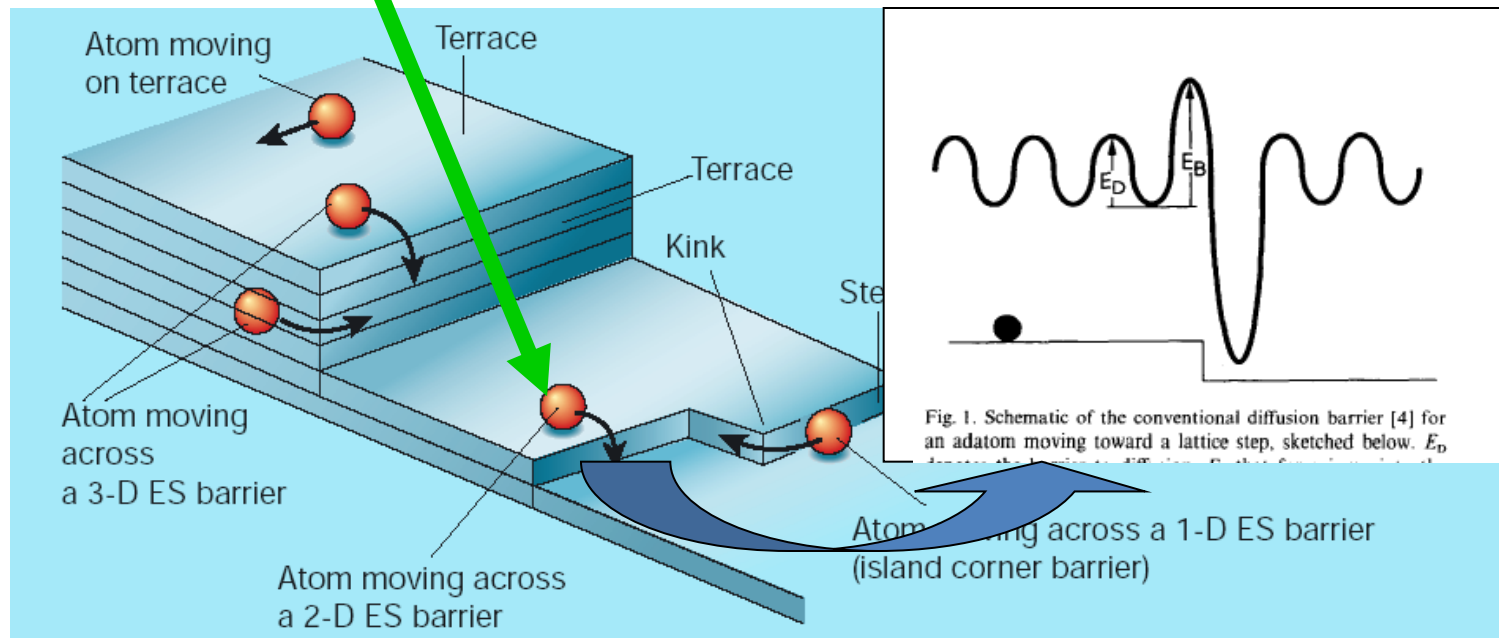


**Terrace-Step-Kink (TSK) Model of Surface:** [Burton, Cabrera, Frank \(1951\)](#)

**STM confirmation:** [Swartzentruber, \*et al.\*, Phys. Rev. Lett. \(1989\)](#)

**Important Atomic Rate Processes:** [Lagally & Zhang, Nature \(2002\)](#)

# Ehrlich-Schwoebel Barrier & Villain Instability



**ES Barrier:** An adatom descending at a step edge encounters a higher potential energy barrier than that for surface diffusion.

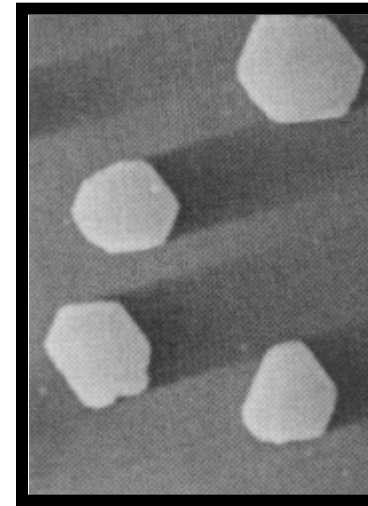
*Ehrlich & Hudda (1966); Schwoebel & Shipsey (1966).*

**Villain Instability:** Growth is unstable if there are no adatom descending events. *Villain (1991).*

# Island Shape Selection in Submonolayer Epitaxy



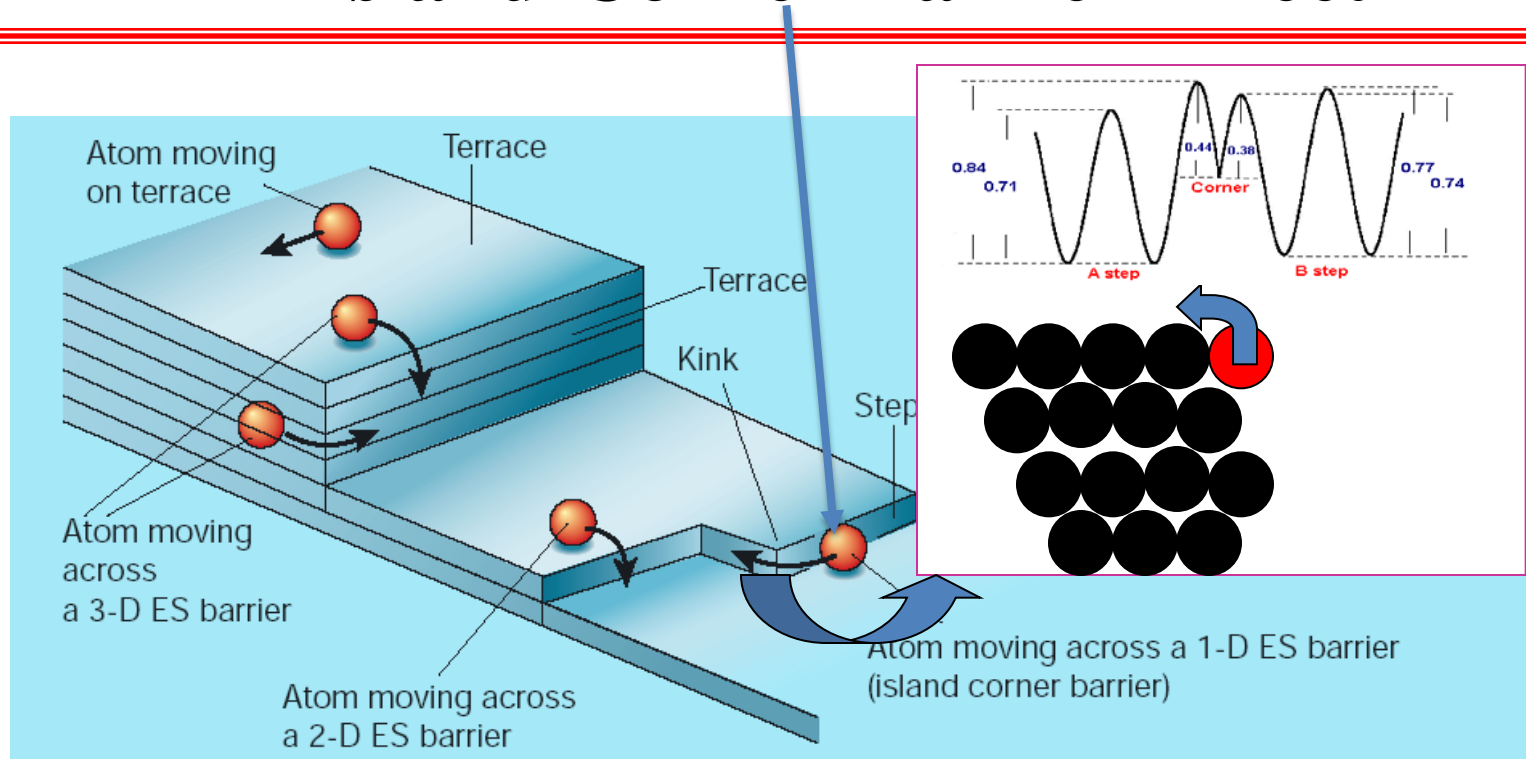
**fractal/dendritic**  
( $N \sim R^d$ ,  $d \sim 1.7$ )



**compact**  
( $N \sim R^d$ ,  $d=2$ )

**Q:** What is the rate-limiting process separating the fractal from compact growth regime?

# Island-Corner Barrier Effect



Without efficient adatom corner crossing, submonolayer growth will lead to the formation of fractal or dendritic islands.

Z. Y. Zhang and Max Lagally, *Science*, 276, 377 (1997);  
Zhong, Tianjiao, Zhang, Z. Y., Lagally, *PRB* 63, 113403 (2001).



# Instabilities in Homoepitaxy vs. Heteroepitaxy

**Homoepitaxy:** All instabilities in nonequilibrium epitaxial growth are **kinetic** in nature.

**Heteroepitaxy:** The growth instabilities can be **thermodynamically** driven, or **kinetically** limited, or **BOTH**, making it conceptually more demanding and challenging.

Even kinetically driven, the growth kinetics can be significantly influenced by **the mismatch-induced strain**.

# Basic Atoms-to-Continuum Method

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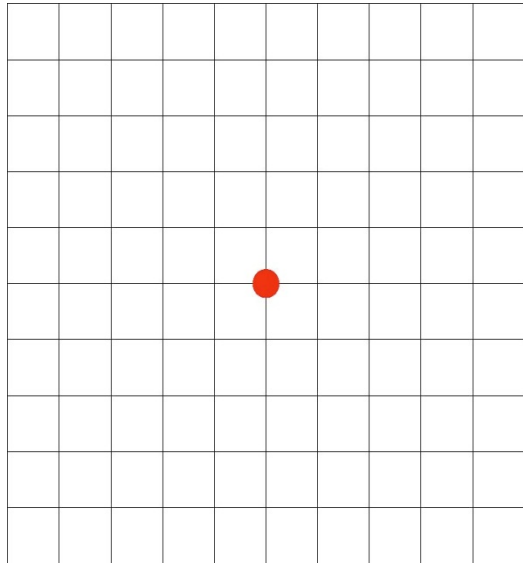
$$P(n, t + \tau) = \frac{1}{2}P[(n - 1)a, t] + \frac{1}{2}P[(n + 1)a, t]$$

$$P(n, t + \tau) = P(n, t) + \frac{\partial P}{\partial t}\tau + \dots$$

$$P[(n \pm 1)a, t] = P(n, t) \pm \frac{\partial P}{\partial x}a + \frac{1}{2}\frac{\partial^2 P}{\partial x^2}a^2 + \dots$$

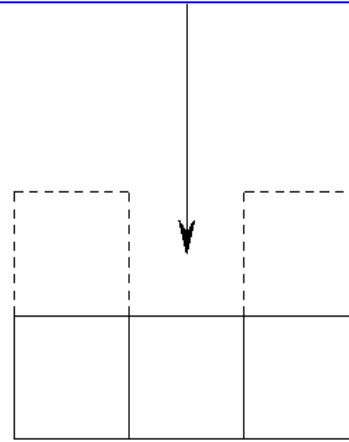
Take  $a \rightarrow 0$  and  $\tau \rightarrow 0$  such that  $a^2/\tau$  is constant:

$$\frac{\partial P}{\partial t} = D\frac{\partial^2 P}{\partial x^2}, \quad D = \frac{a^2}{2\tau}$$

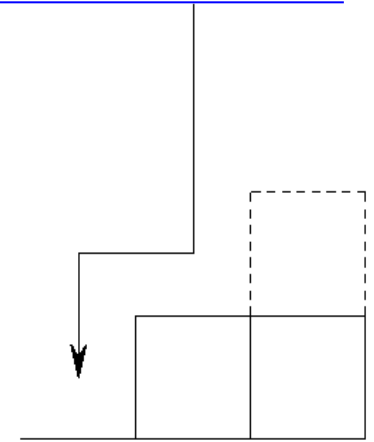


# Edwards–Wilkinson Model

A particle that arrived on top of the column at  $x$  sticks at the lowest column among the nearest neighbours,  $x$  and  $x + 1$  or  $x - 1$ .

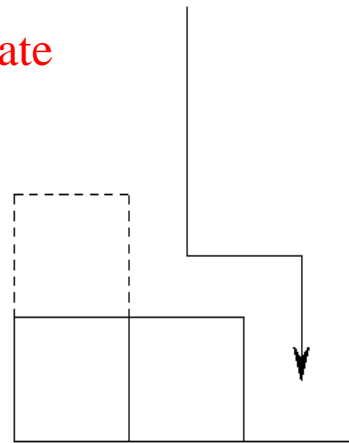
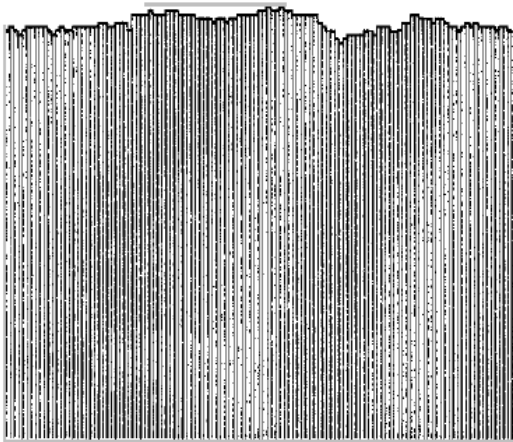


(a)

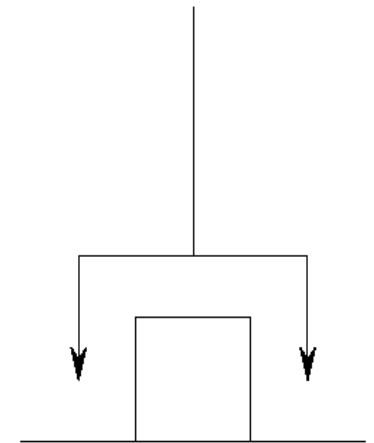


(b)

Surface configuration in the steady state



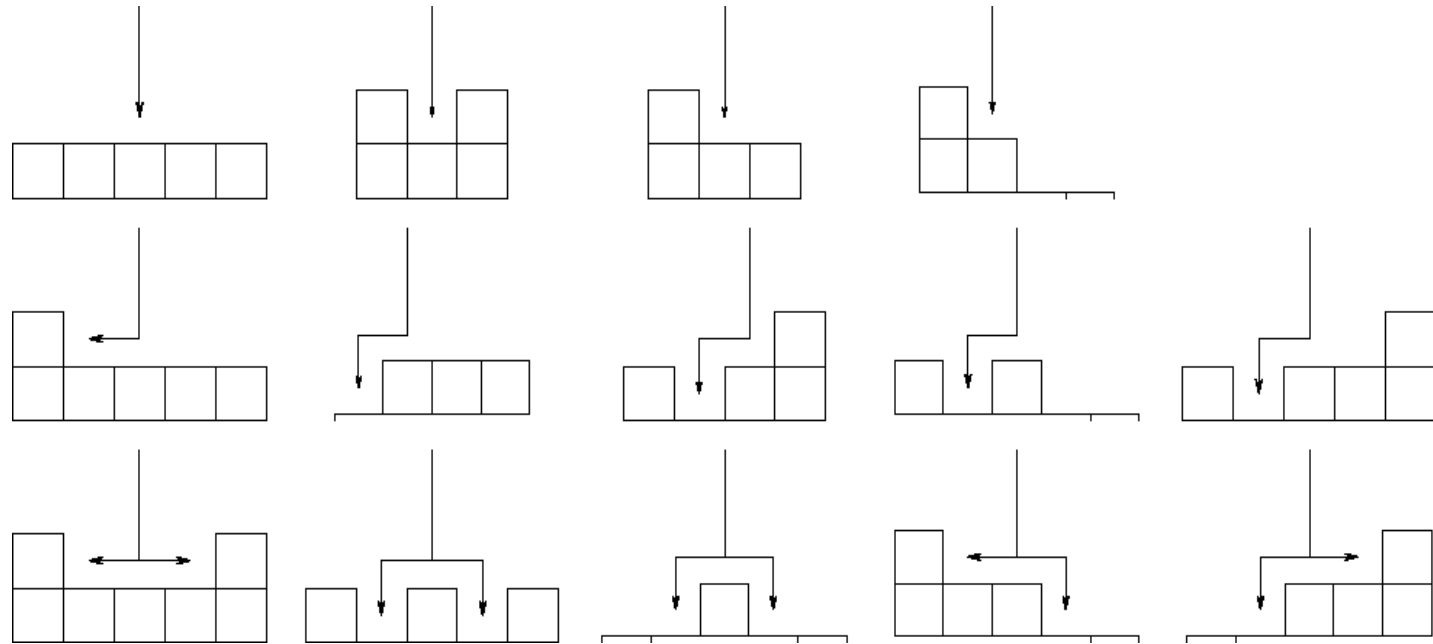
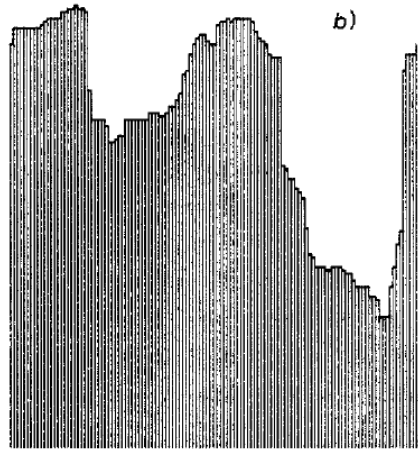
(c)



(d)

# The Wolf-Villain Model

Surface configuration in the steady state



A particle that arrived chooses the column on top of which it touches the most occupied sites (*i.e.* the most occupied neighbors).

This is intended to simulate surface diffusion at not too high temperatures where the particles move only a short distance to find a favorable growth site before other particles are deposited on top of them.

Wolf and Villain, *Europhys. Lett.* **13**, 389 (1990)  
Clarke and Vvedensky, *Phys. Rev. B* **37**, 6559 (1988)

# **Main Focus of Those Statistical Modeling**

**Gives Scaling Properties & Universality (but  
minimal touch with real systems)**

# Thin Film Growth on Flat or Vicinal Substrates

## Dynamics of Strained Film Growth on Vicinal Substrates

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*Staff Scientist*

*Oak Ridge National Laboratory*

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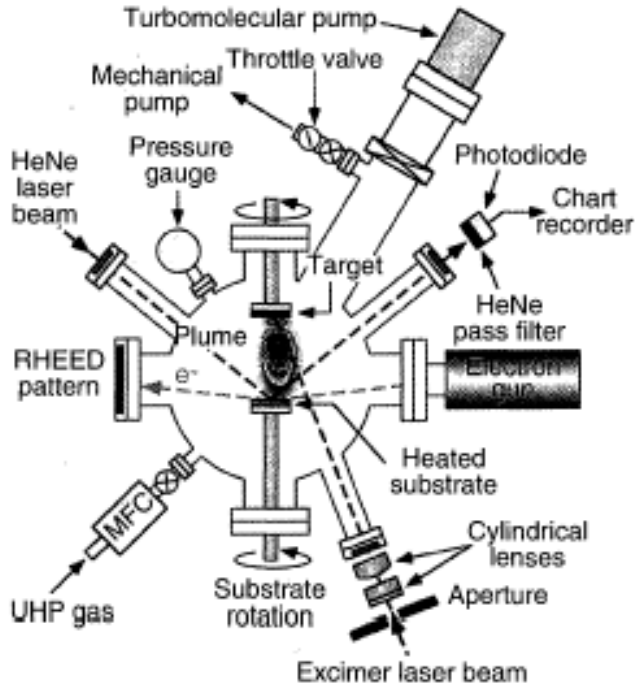
[myoon@ornl.gov](mailto:myoon@ornl.gov)

**In Collaboration with**

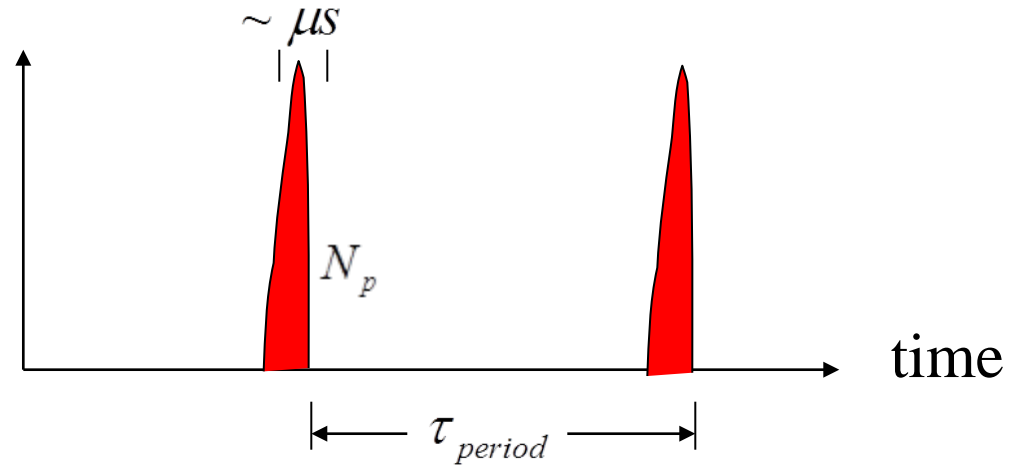
**Experiment: Ho Nyung Lee, Hans Christen (ORNL)**

**Theory: Wei Hong, Zhigang Suo (Harvard), Zhenyu Zhang (ORNL, UT)**

# Pulse Laser Deposition (PLD)



Flux (ML/s)



**# of monolayers per pulse**  $N_p = 0.006 \text{ ML}$

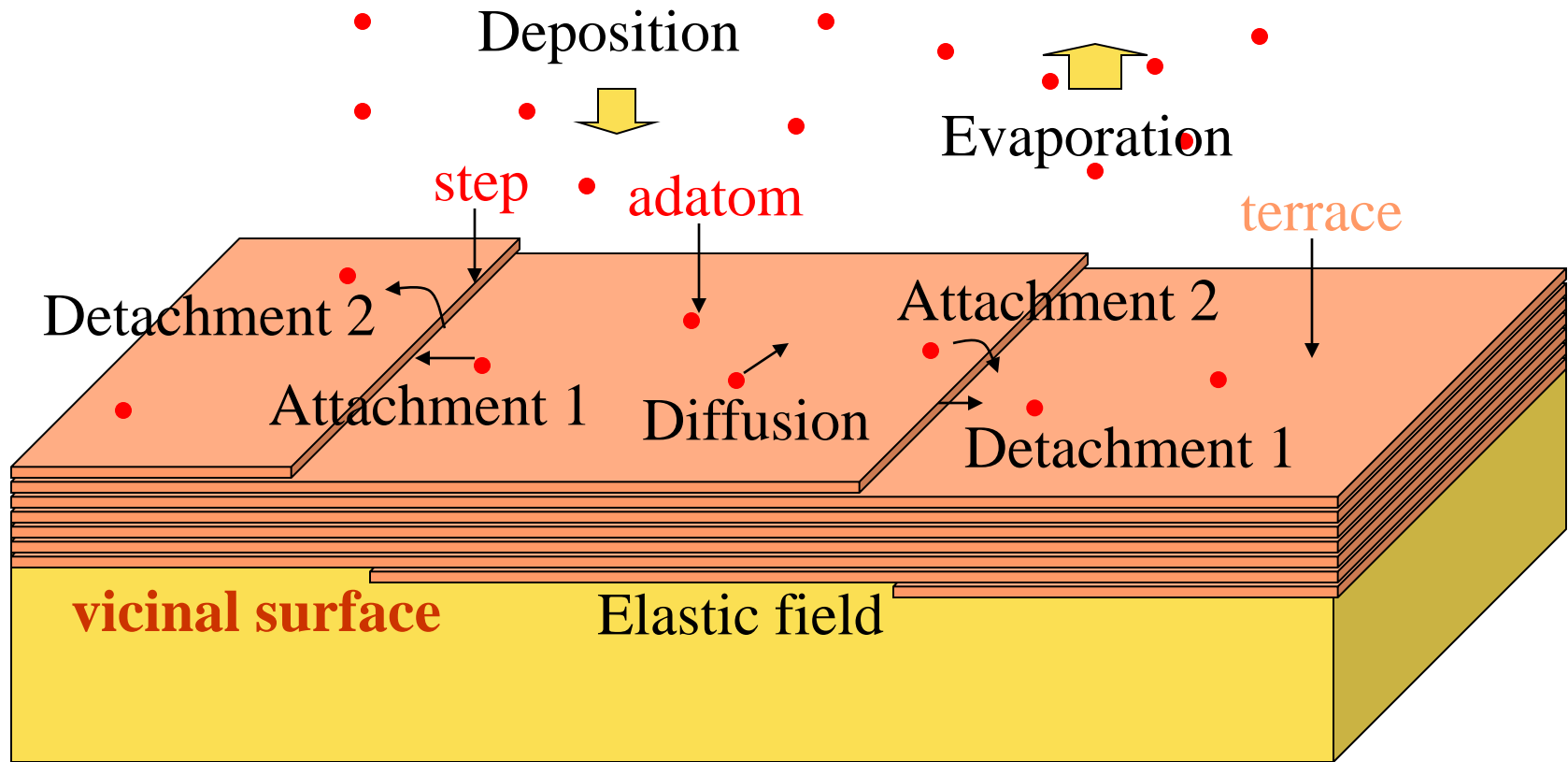
**Time between two pulses**  $\tau_{\text{period}} = 0.1 \text{ s}$

**Average flux**  $F = \frac{N_p}{\tau_{\text{period}}} \text{ (ML/s)}$

Lowndes *et al.*, Science **273**,  
898 (1996)

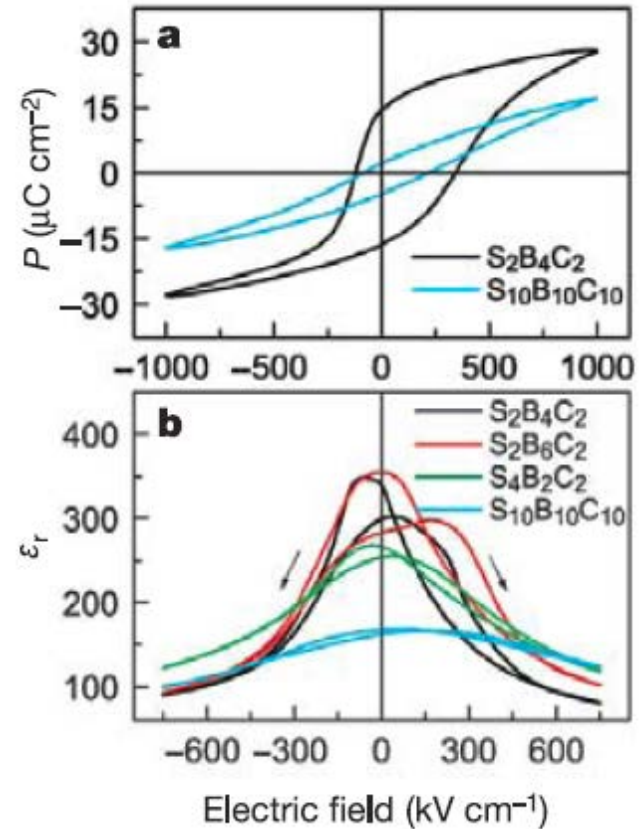
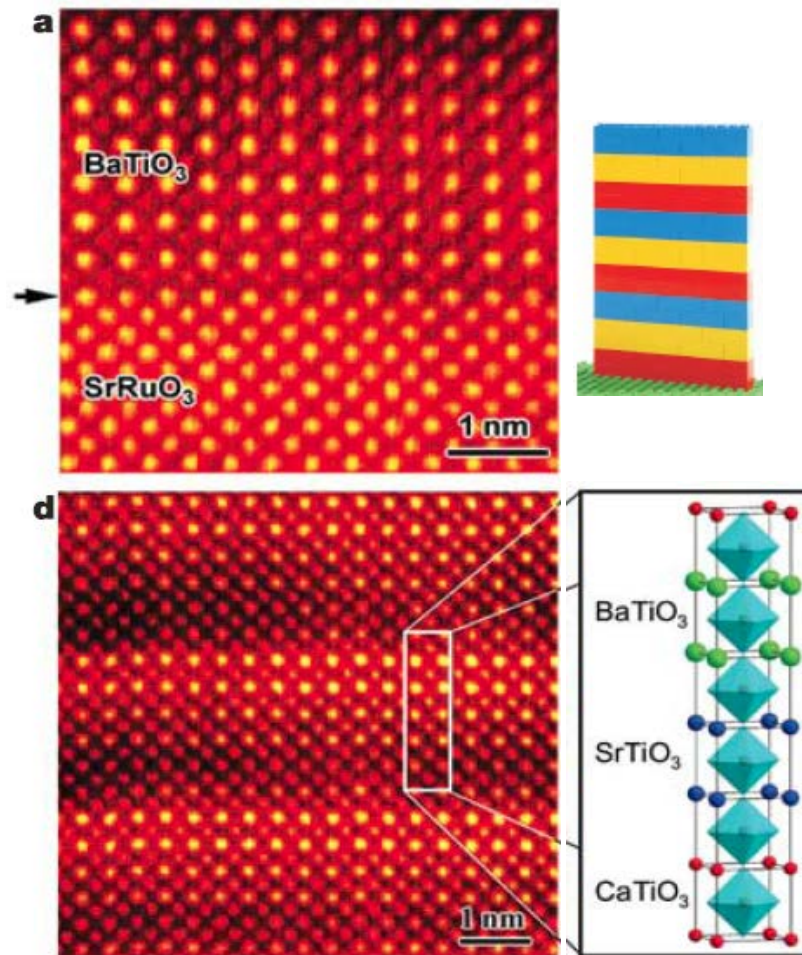


# Epitaxy on a Vicinal Surface





# Superlattice of Ferroelectric Oxides

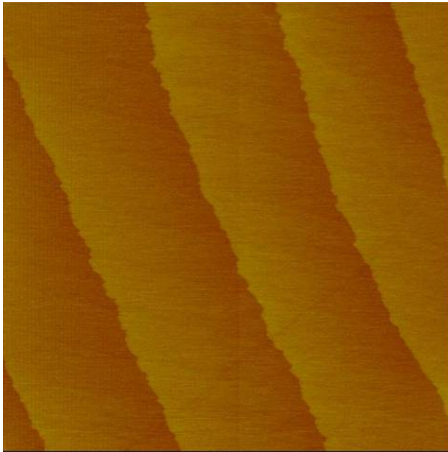
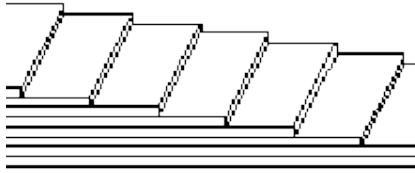


Lee *et al.*, Nature **433**, 395 (2005)

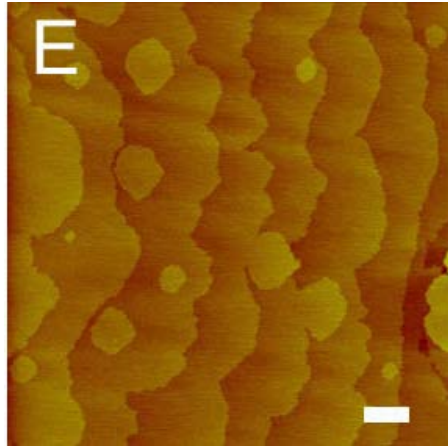
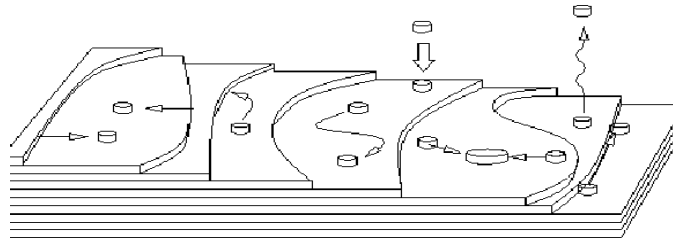


# Growth Modes

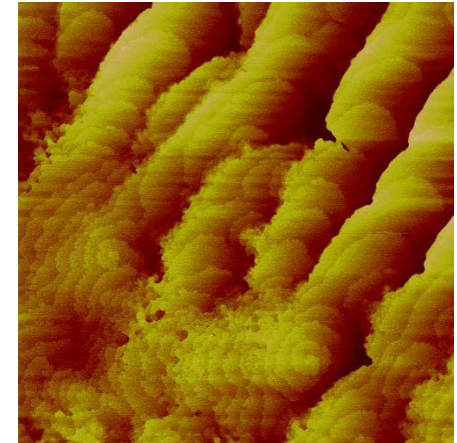
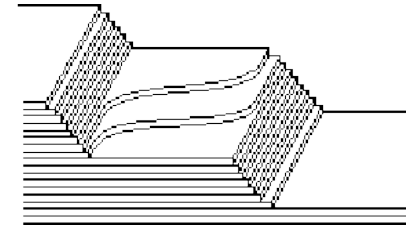
## Step Flow



## Island Formation



## Step Bunching



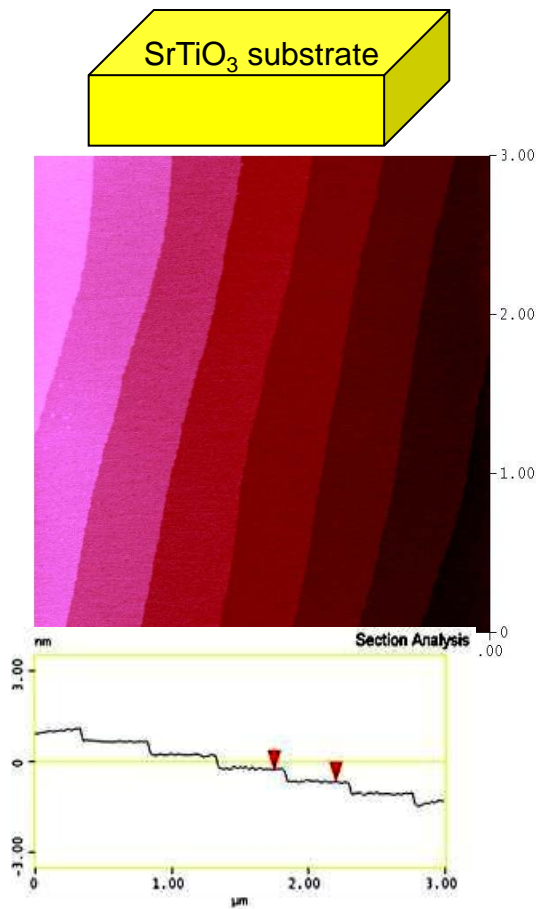
Step flow is preferred

**But how to achieve step flow?**

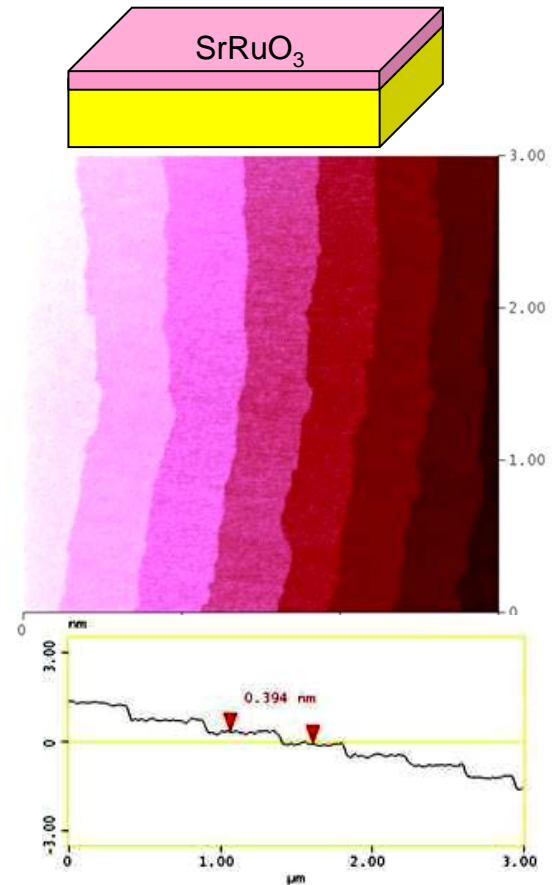
for applications that require defect-free, atomically smooth films.



# Experiments: SrRuO<sub>3</sub> on SrTiO<sub>3</sub> Growth by PLD



$$a_{STO} = 0.391nm$$



$$\epsilon_m = 0.6\% \quad a_{SRO} = 0.393nm$$

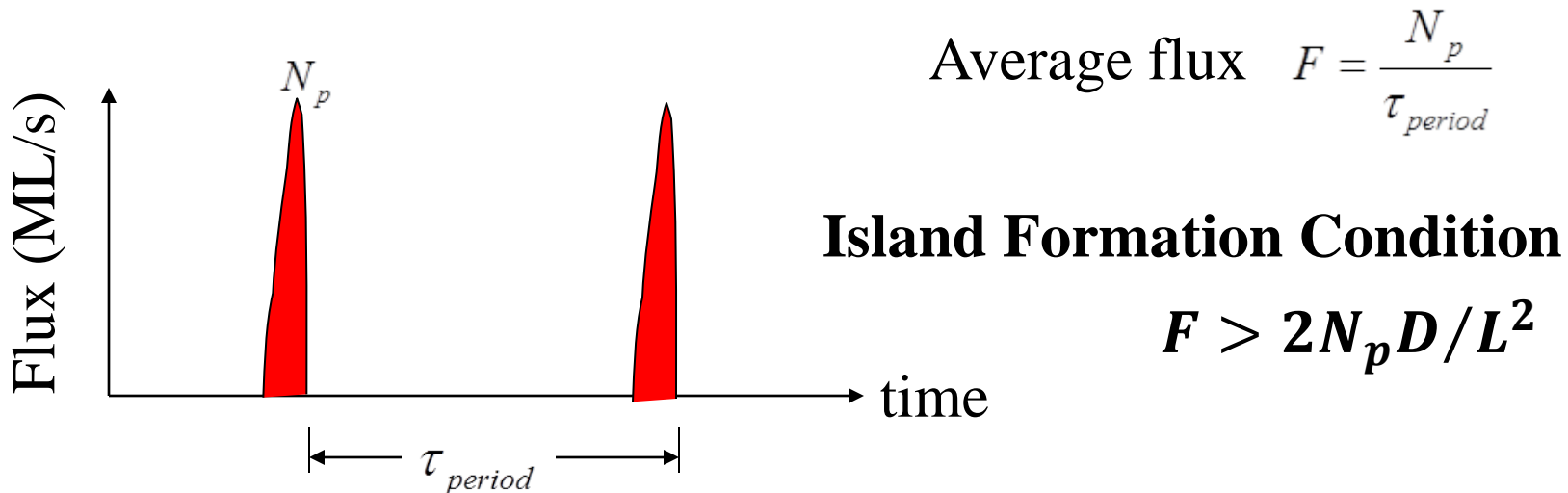
*Experiments: H. Lee et al.*

## X-ray diffraction:

SRO films were fully strained to match the STO substrates.

# Island Formation in PLD

- Temperature is high. In each deposition pulse, islands may (or may not) form, but they are unstable.
- Time to evacuate adatoms from a terrace  $\tau_{life} = L^2 / 2D$
- Islands will form when  $\tau_{life} > \tau_{period}$



- **Kinetic effect**

- Asymmetric attachment promotes step flow.

Schwoebel, J. Appl. Phys. **40**, 614 (1969)

- **Energetic effect**

- Elastic energy promotes step bunching.

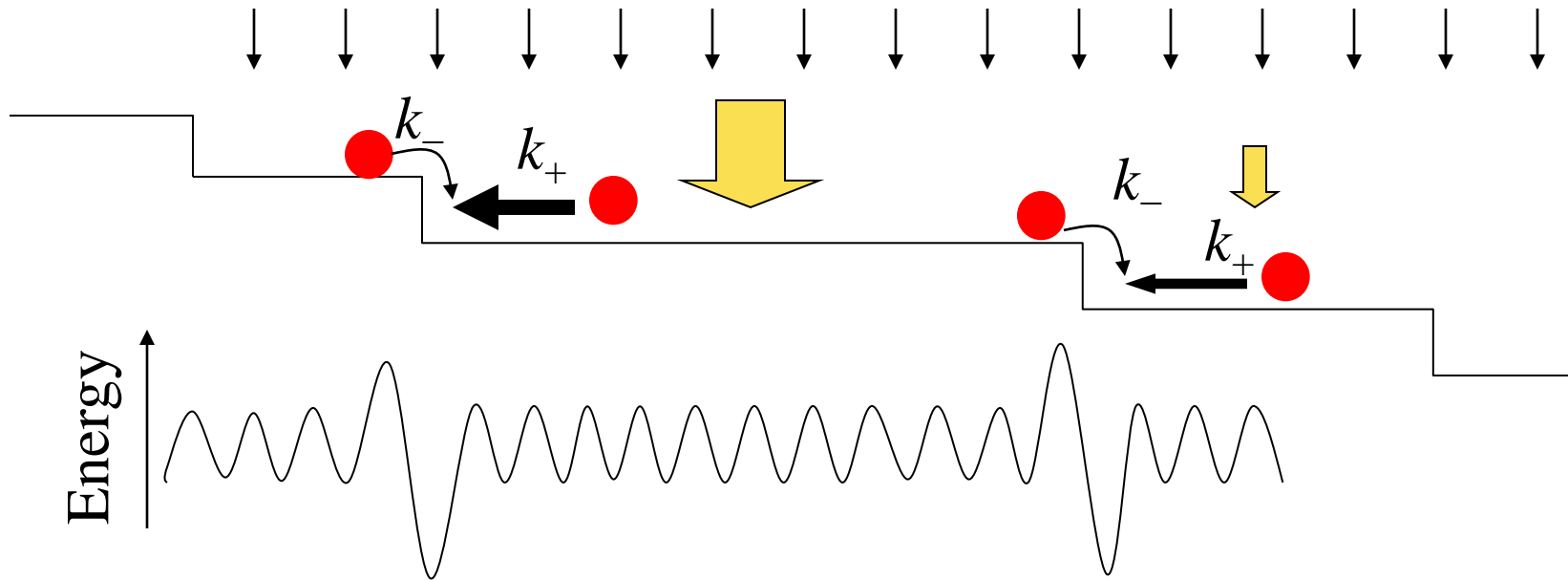
Tersoff et al, PRL **75**, 2730 (1995)

**Step flow prevails when deposition flux is high enough.**





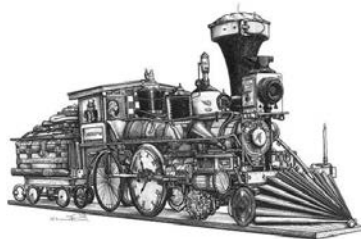
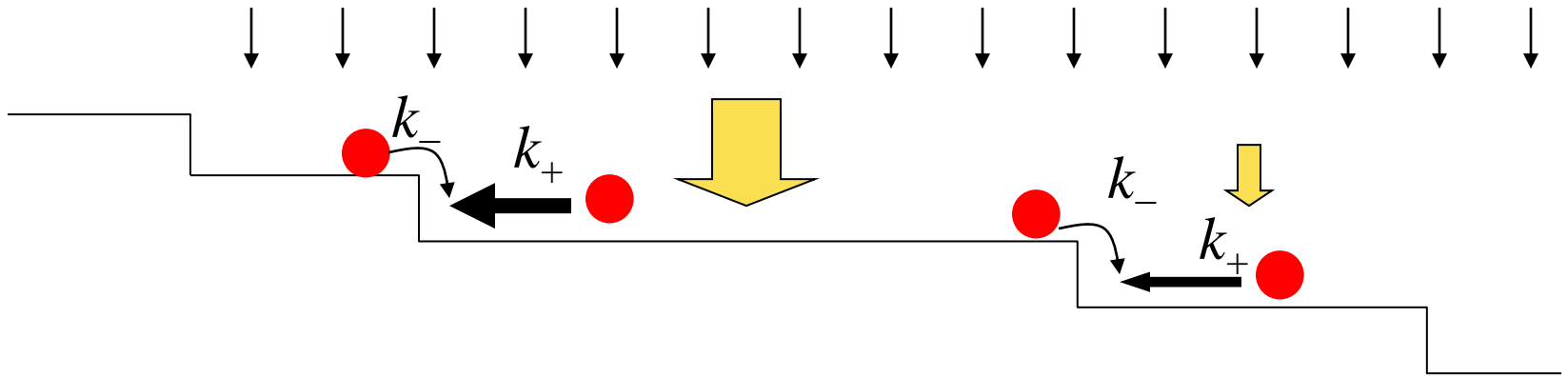
# Asymmetric attachment promotes step-flow



- Ehrlich-Schwoebel barrier [Schwoebel, J. Appl. Phys. 40, 614 \(1969\)](#)
- Asymmetric attachment:  $k_+ \gg k_-$
- **Schwoebel effect**: during deposition, a large terrace becomes small, and a small terrace becomes large.



# Schwoebel effect



LATE  
4 min

Harvard

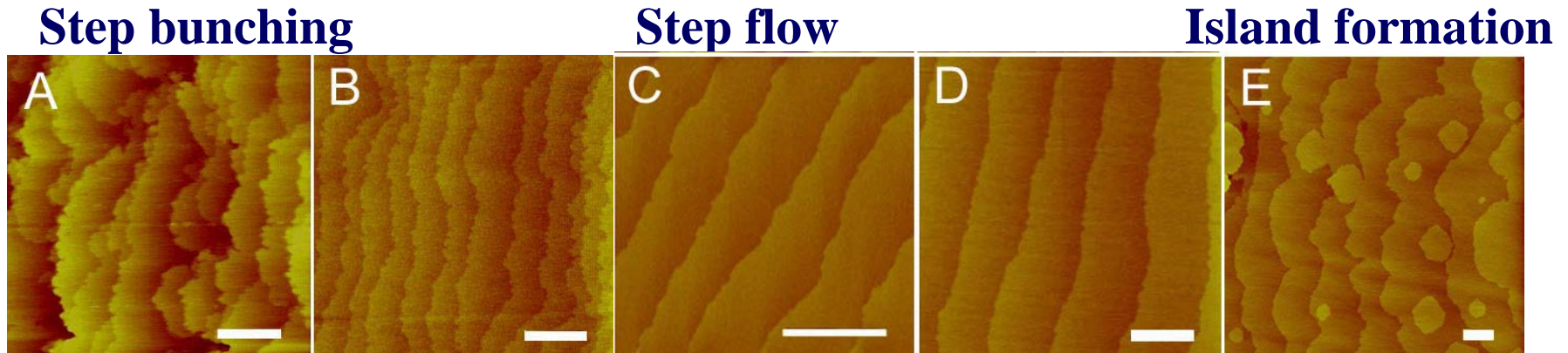


# Questions to be addressed

- ❖ How to stabilize step flow?
- ❖ Can we determine the stability of step flow  
in the early stage before step bunching occurs?
- ❖ Is there any thickness dependence in the step flow?



# How to Stabilize Step Flow?



AFM images of SrRuO<sub>3</sub>/SrTiO<sub>3</sub>

*Images from [W. Hong et al., Phys. Rev. Lett. 95, 095501 (2005)]*

**Thus, in order to achieve step flow:**

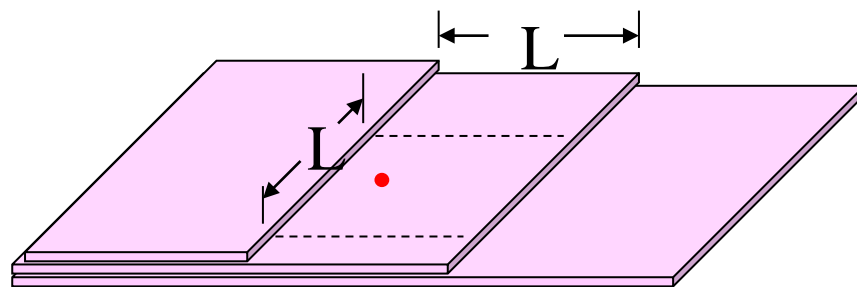
- ❖ **Avert island formation**
- ❖ **Avert step bunching**

# Avert Island Formation In Molecular Beam Epitaxy (MBE) condition

Homogeneous deposition of adatoms

**Temperature is Low**

Once an island forms, it is **stable**



Life time of adatoms (time for an adatom reach a step):  $\tau_{life} = L^2 / 2D$

Landing time (time between two consecutive incident adatoms to land in an area of  $L^2$ ):

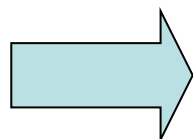
$$\tau_{land} = \left(\frac{a}{L}\right)^2 \frac{1}{F}$$

**To avert island formation:**  $\tau_{life} < \tau_{land}$

$D$ : diffusivity,

$F$ : flux

$a$ : lattice constant

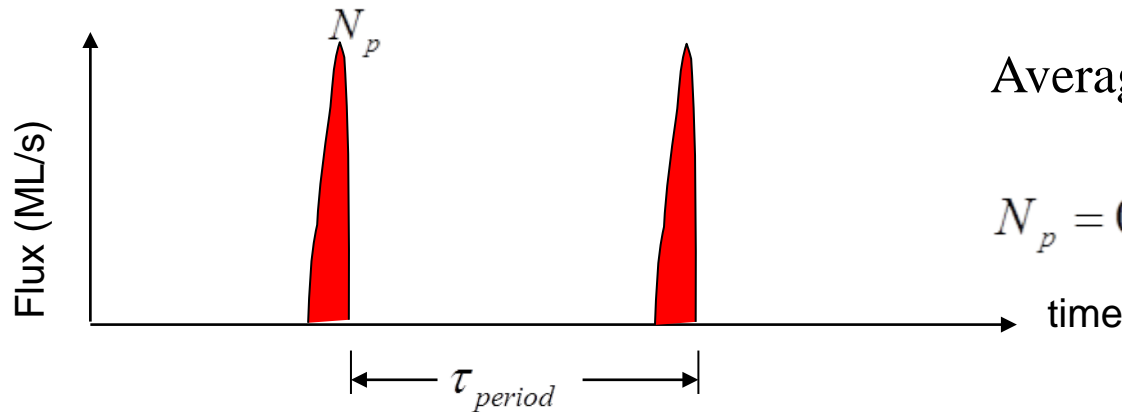

 $F < 2Da^2 / L^4$

# Avert Island Formation in Pulsed Laser Deposition Condition

Each pulse deposits a large number of atoms

**Temperature is high**

In each deposition pulse, islands may form, but they are **unstable**



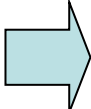
Average flux  $F = \frac{N_p}{\tau_{period}}$

$N_p = 0.006\text{ML}$      $\tau_{period} = 0.1\text{s}$

Life time of adatoms (time for an adatom reach a step):  $\tau_{life} = L^2 / 2D'$

Landing time (period between pulses):  $\tau_{period} = \frac{N_p}{F}$

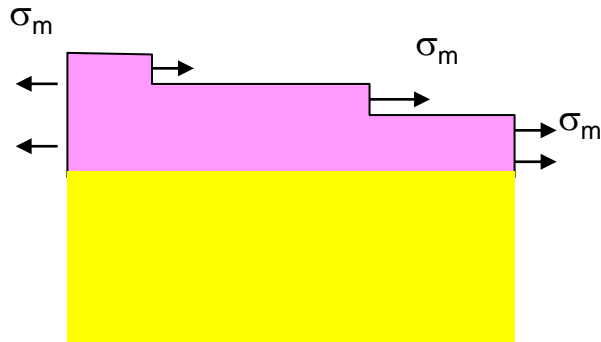
To avert island formation:  $\tau_{life} < \tau_{period}$


 $F < 2N_p D / L^2$

$D'$ : effective diffusivity

# Avert Step Bunching

## ❖ Heteroepitaxy: Elastic Energy



### • Energetics effect:

On a strained surface, steps **attract** each other

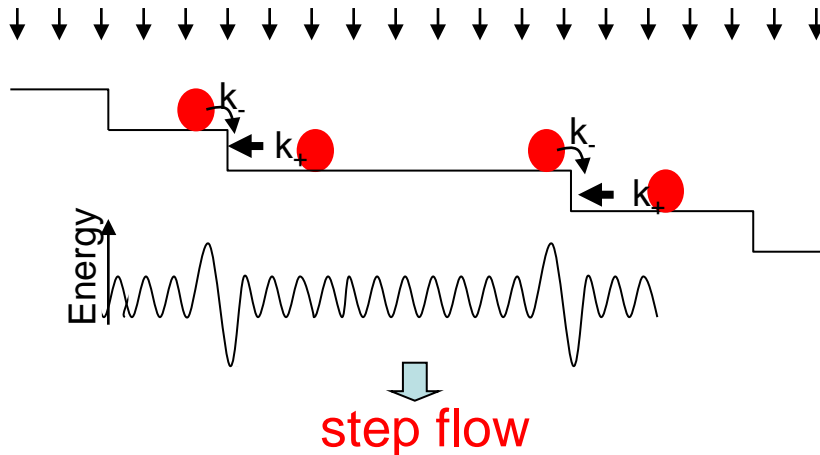
[Marchenko, JEPT 54, 605 (1981)]

[Tersoff et al., PRL (1995)75, 2730]



**step bunching**

## ❖ On a vicinal substrate: Ehrlich-Schwoebel barrier



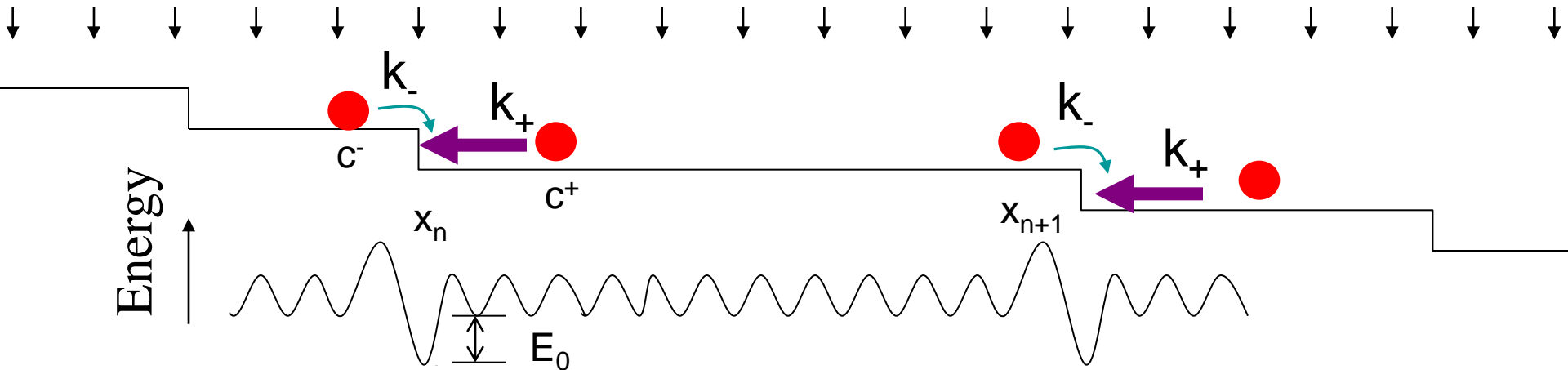
### • Kinetic effect:

Asymmetric attachment:  $k_+ \gg k_-$

[Schwoebel, J. Appl. Phys. 40, 614 (1969)]

## ❖ Competition: Energetics vs. Kinetics

# Evolution of Adatoms & Steps



- Equilibrium coverage of adatoms under the step-step elastic interactions  $f_n(x_n)$ :

$$c_{eq}(x_n) = \exp\left(-\frac{E_0 + Af_n}{k_B T}\right) \quad f_n = \alpha \sum_{m=\pm 1}^{\pm\infty} \frac{1}{x_{n+m} - x_n} \quad [Tersoff et al., PRL (1995)75, 2730]$$

- Adatom diffusion flux:

$$-D \frac{\partial c}{\partial x} = k_- [c^- - c_{eq}] \quad D \frac{\partial c}{\partial x} = k_+ [c^+ - c_{eq}]$$

$$c(x_n^-) \neq c(x_n^+) \neq c_{eq}(x_n)$$

- Step velocity:  $\frac{dx_n}{dt} = k_- [c(x_n^-) - c_{eq}(x_n)] - k_+ [c(x_n^+) - c_{eq}(x_n)]$

# Equation of Motion

- Quasi-steady state approximation:**

Low flux

Steps move slowly within the time to approach steady state distribution

$$\frac{\partial c}{\partial t} = D \frac{\partial^2 c}{\partial x^2} + F(t) \sim 0 \quad \Rightarrow \quad c(x) = -\frac{F}{2D}(x - x_n)^2 + \beta_1(x - x_n) + \beta_0$$

- Equation of motion of the step:**

$$\frac{dx_n}{dt} = k_- \left[ c(x_n^-) - c_{eq}(x_n) \right] - k_+ \left[ c(x_n^+) - c_{eq}(x_n) \right]$$

With proper boundary conditions

$$\frac{dx_n}{dt} = \frac{-Q_{n-1} + Fl_{n-1} \left( \frac{1}{k_-} + \frac{l_{n-1}}{2D} \right)}{\frac{1}{k_-} + \frac{1}{k_+} + \frac{l_{n-1}}{D}} + \frac{Q_n + Fl_n \left( \frac{1}{k_-} + \frac{l_n}{2D} \right)}{\frac{1}{k_-} + \frac{1}{k_+} + \frac{l_n}{D}} \quad \begin{aligned} Q_n &= c_{eq}(x_{n+1}) - c_{eq}(x_n) \\ l_n &= x_{n+1} - x_n \end{aligned}$$

# Avert Step Bunching: Linear Stability Analysis

- **Perturbation:**  $x_{n+1} - x_n = L + q(t)\exp(iKn)$

$$dq / dt = \Omega q \quad \text{where}$$

$$\Omega = \frac{1 - \cos K}{\frac{1}{k_-} + \frac{1}{k_+} + \frac{L}{D}} \left[ \frac{\alpha_1 A c_0 S}{k_B T L^2} - F \frac{\frac{1}{(k_-)^2} - \frac{1}{(k_+)^2}}{\frac{1}{k_-} + \frac{1}{k_+} + \frac{L}{D}} \right] + iF \sin K$$

**Re  $\Omega > 0$  unstable**  
**Re  $\Omega < 0$  stable**

- **Step flow stability condition:**  $\text{Re } \Omega < 0$   
 (  $k_+ \gg k_-$  )

$$\Rightarrow F > \frac{\pi^2 \alpha_1 A c_0 k_-}{k_B T L^2} \left( 1 + \frac{L k_-}{D} \right)$$

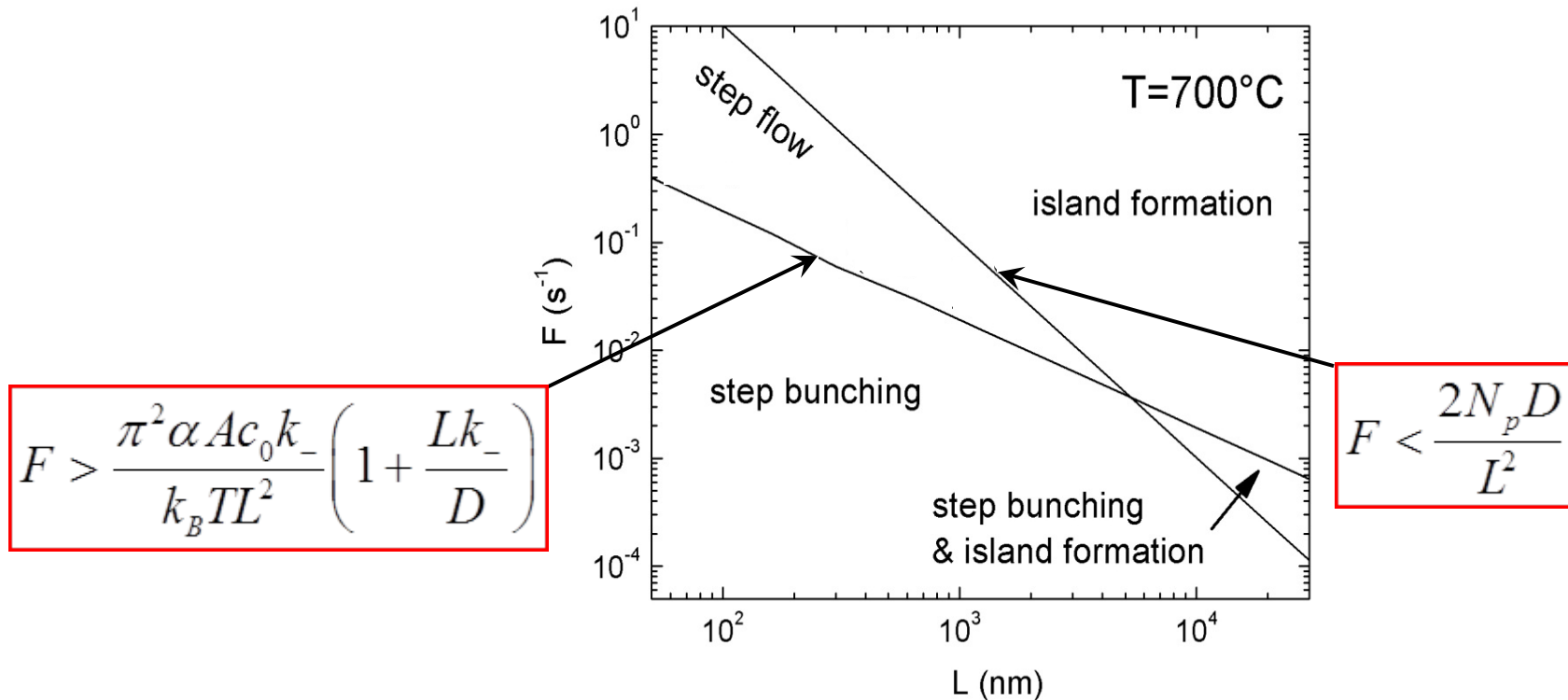
$$c_0 = e^{-\beta E_0}, D = D_0 e^{-\beta E_D}, k_- = \frac{D}{a} e^{-\beta E_{ES}}, \alpha_1 = \alpha_1(\varepsilon, Y, \nu)$$

$E_{ES}$  : ES barrier  $E_D$ : diffusion barrier  $E_0$ : formation energy of an adatom

$D$  : adatom diffusion coefficient  $L$  : average terrace width

$\alpha_1$  : elastic interaction parameter

# Growth Mode Diagram

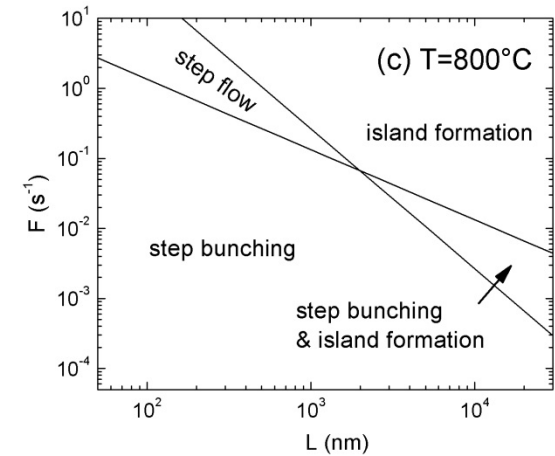
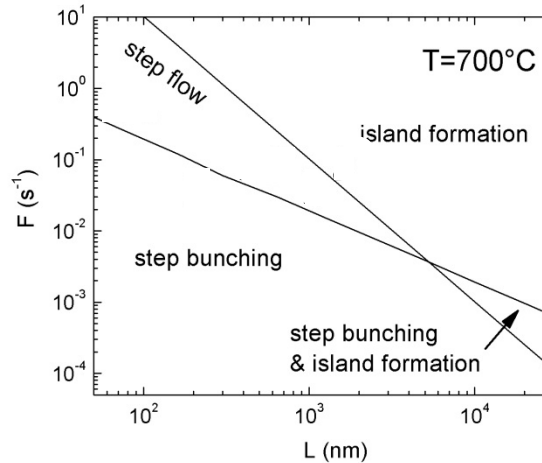
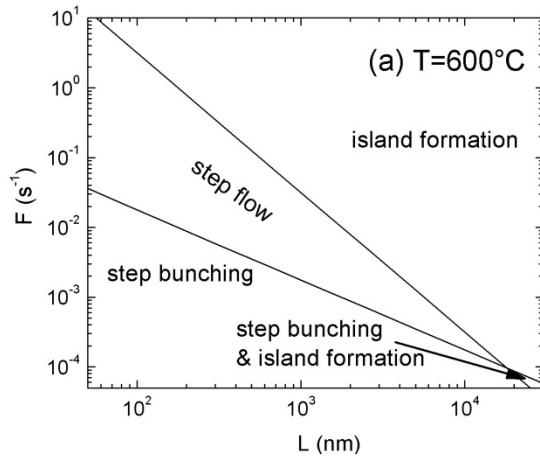


- New growth mode: concurrent step bunching and island formation
- Existence of persistent step flow



# Growth Mode Diagram (*continued*)

## ❖ Temperature Dependence



### To avert step bunching

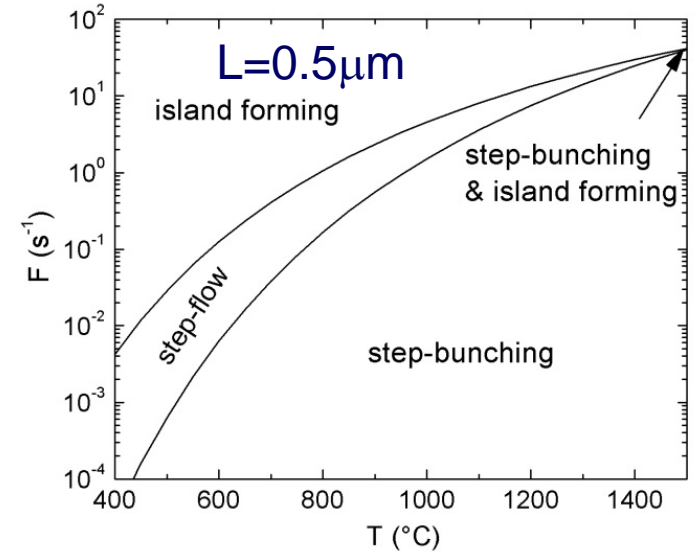
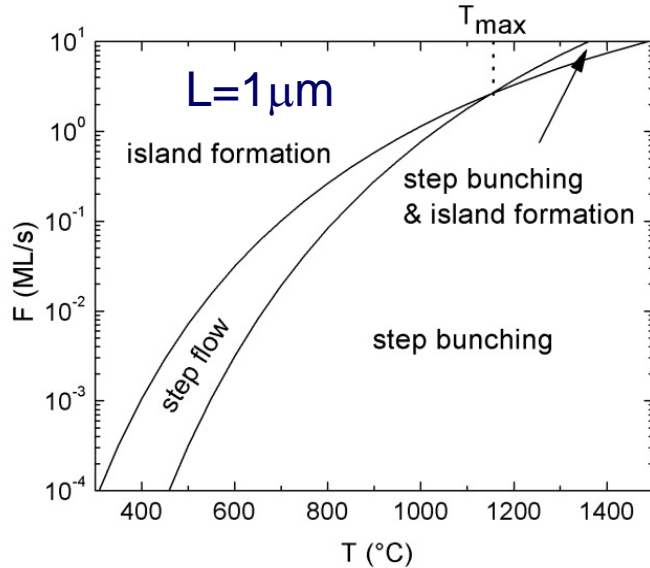
$$F > \frac{\pi^2 \alpha A c_0 k_-}{k_B T L^2} \left( 1 + \frac{L k_-}{D} \right) \sim \begin{cases} \frac{1}{L^2} \exp\left( -\frac{E_0 + E_D + E_{ES}}{k_B T} \right) & \text{small } L \\ \frac{1}{L} \exp\left( -\frac{E_0 + E_D + 2E_{ES}}{k_B T} \right) & \text{large } L \end{cases}$$

### To avert island formation

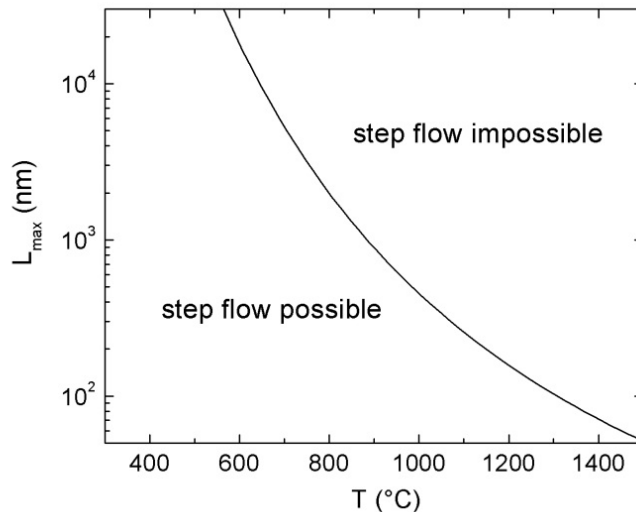
$$F < \frac{2N_p D}{L^2} \sim \frac{1}{L^2} \exp\left( -\frac{E_D}{k_B T} \right)$$

# Growth Mode Diagram (*continued*)

## ❖ Flux vs Temperature



## ❖ Maximum Terrace Width

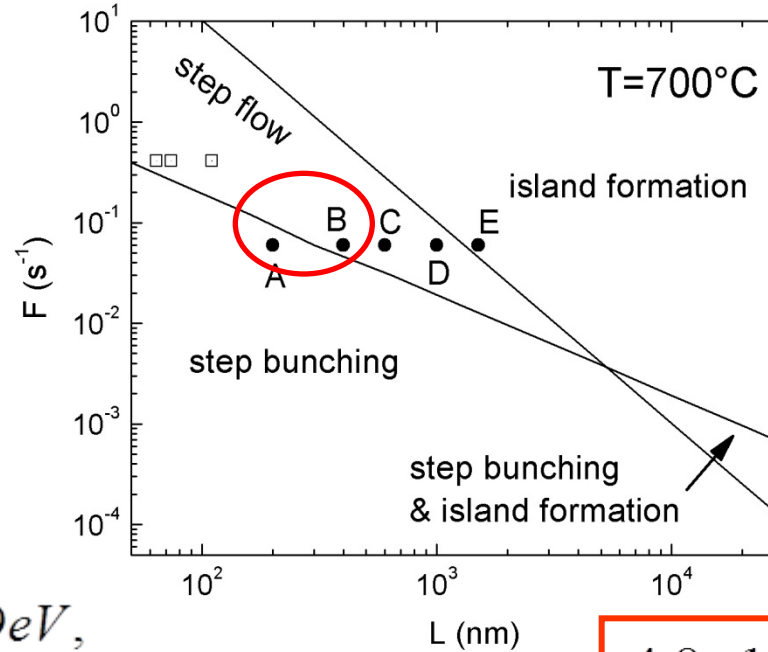


$$L_{\text{max}} = \frac{D}{k_-} \left( \frac{2k_B T N_p D}{\pi^2 A \alpha_1 c_0 k_-} - 1 \right)$$

# Experiments & Theory

[Hong et al., Phys. Rev. Lett. 95, 095501(2005)]

$F=0.06$  ML/s



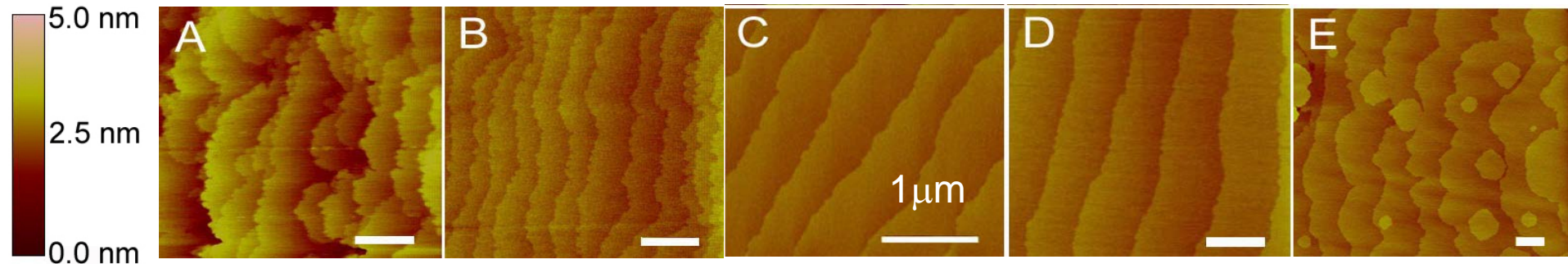
Parameters found:

$$E_0 = 0.8\text{eV}, E_D = 0.9\text{eV},$$

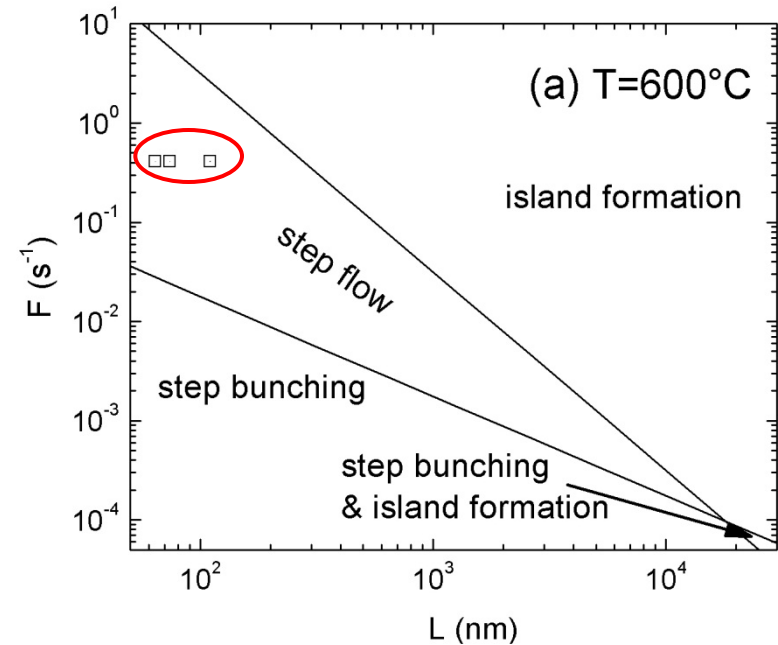
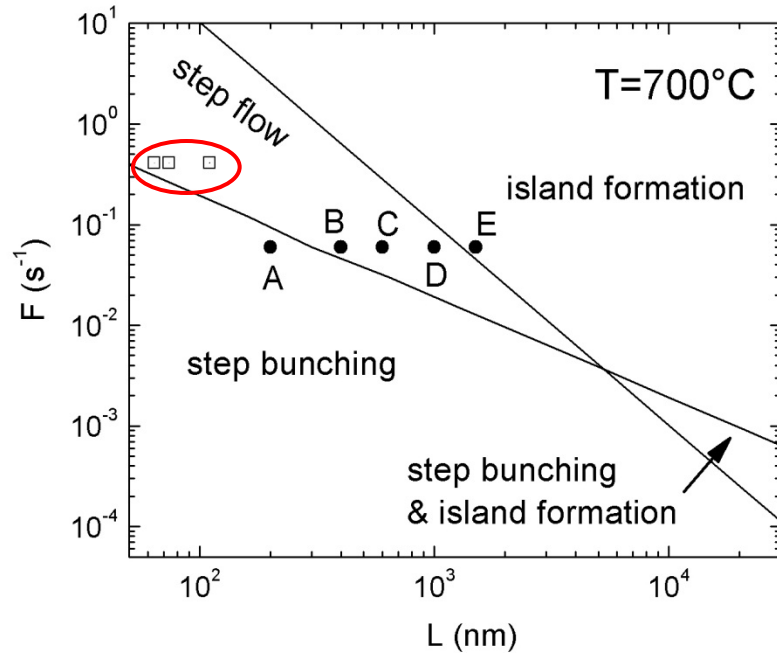
$$E_{ES} = 0.1\text{eV}, D_0 = 2.3 \times 10^{11} \text{ nm}^2 / \text{s}$$

$$4.8 \times 10^6 < D < 1.1 \times 10^7 \text{ nm}^2 / \text{s}$$

$$1.02 < E_0 + 2E_{ES} < 1.15\text{eV}$$



# Experiments & Theory (*continued*)



Experimental data points:

J. Choi *et al.*, Appl. Phys. Lett. **79**, 1447 (2001)

G. Rijnders *et al.*, Appl. Phys. Lett. **84**, 505 (2004)

# Can We Determine the Stability of the Step Flow in the Early Stage?

## ❖ Time evolution of step position:

$$dx_n / dt = V \left[ (x_{n+1} - x_n), (x_{n+2} - x_n), \dots, (x_{n-1} - x_n), (x_{n-2} - x_n), \dots \right]$$

$V$ : translational symmetry in the system is conserved

## ❖ Step-width evolution:

$$d\delta_n / dt = \sum_{m=\pm 1}^{\pm\infty} (\delta_{n+m} - \delta_n) \partial V / \partial r_m \quad \delta_n(t) = x_{n+1}(t) - x_n(t) - L$$

## ❖ Fourier transformation: $\hat{\delta}(K, t) = \sum_{n=-\infty}^{+\infty} \delta_n(t) \exp(-iKn)$      $\hat{\delta}(K, t) = \hat{\delta}(K, t_0) \exp[\Omega(K)(t - t_0)]$

$$\longrightarrow \partial \hat{\delta} / \partial t = \Omega \hat{\delta} \quad \Omega(K) = \sum_{m=\pm 1}^{\pm\infty} [\exp(iKm) - 1] \partial V / \partial r_m$$

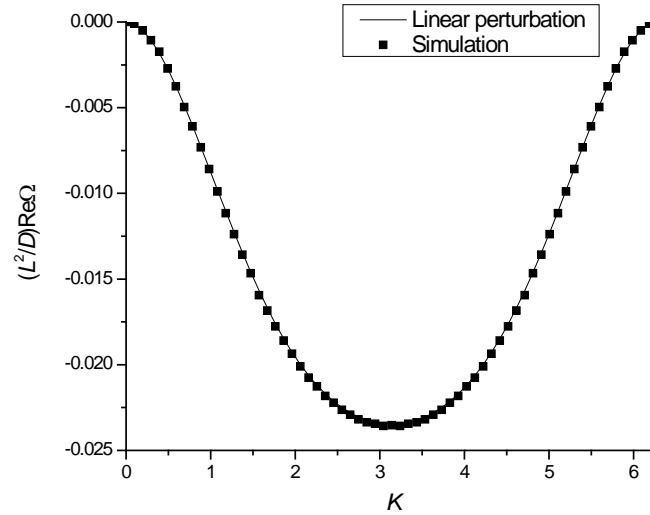
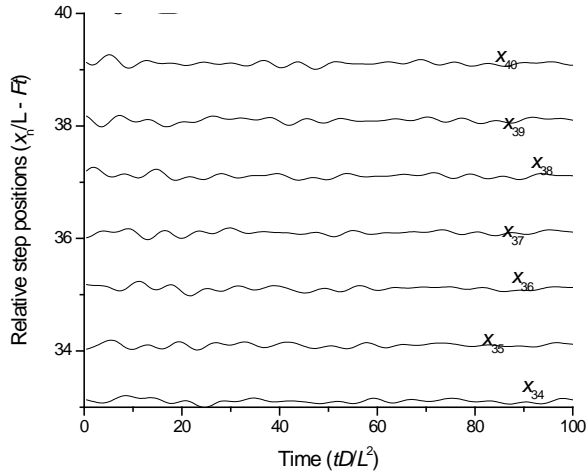
## ❖ Determination of the stability of the step flow: $\text{Re}[\Omega] = \frac{1}{t - t_0} \ln \left| \frac{\hat{\delta}(K, t)}{\hat{\delta}(K, t_0)} \right|$

Step positions at two different times:

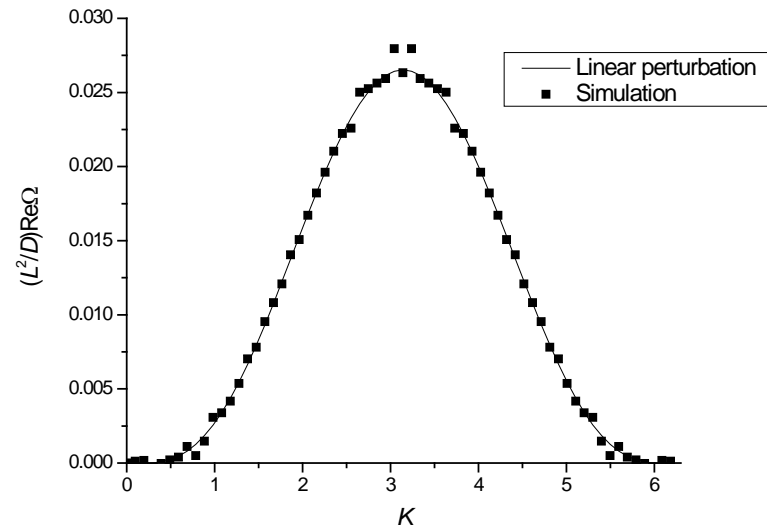
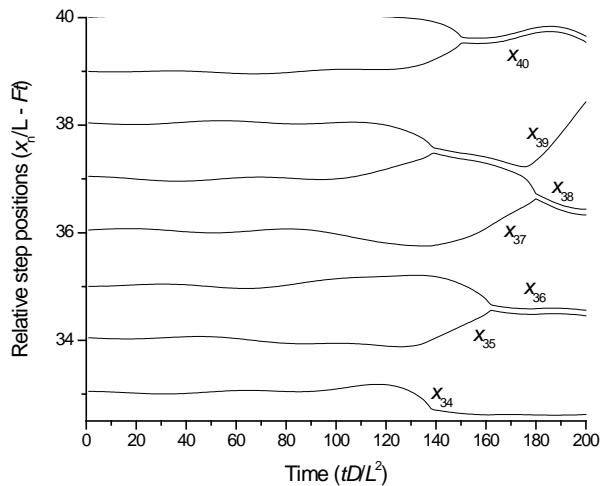
System parameters and growth mode can be determined!

# Fourier analysis of the dynamics

## ❖ Stable: Step Flow (high flux)



## ❖ Unstable: Step Bunching (low flux)



# Any Thickness Dependence in Step Flow?

Evolution of steps in the “step bunching” region

## ❖ Time evolution of the step position

$$x_n = L(n + Ft) + \Delta(t) \sin(2\pi n / N), \quad \Delta(t) = \Delta(0)e^{\Omega t}$$

$$\Rightarrow \text{Re}\Omega = \frac{1}{t - t_0} \ln \left| \frac{\hat{\delta}(K, t)}{\hat{\delta}(K, t_0)} \right| \approx \frac{\alpha_1 A c_0 D}{k_B T L^3} \left[ 1 - \frac{L}{L^*} \frac{\pi^2}{K(2\pi - K)} \right] (1 - \cos K)$$

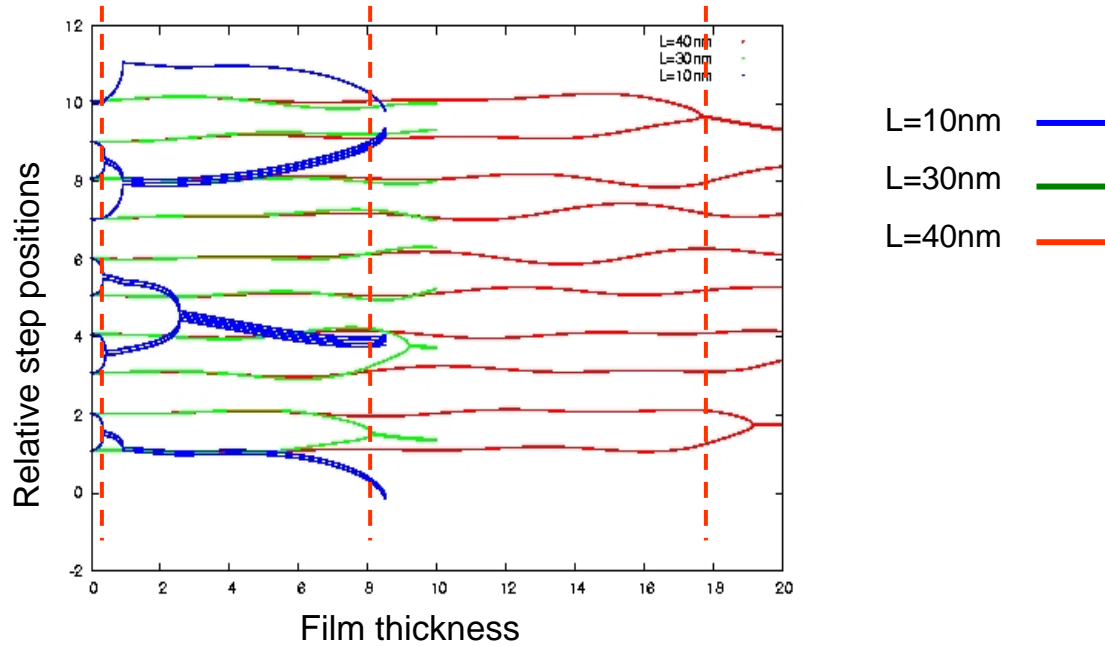
Maximum at  $K = \pi$

$L^*$ : critical terrace width above which a persistent step flow occurs

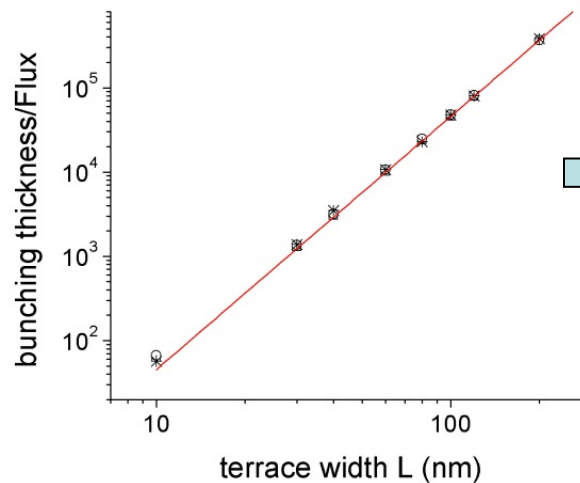
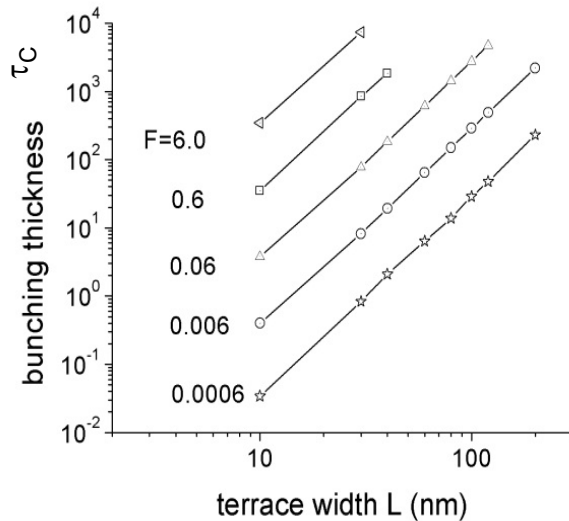
$$\frac{\tau_c}{F} \approx C \frac{k_B T L^3}{\alpha_1 A c_0 D} \left( 1 - \frac{L}{L^*} \right)^{-1}$$

For a film thickness  $t < t_c$ : step-flow occurs in the step bunching regime

# Numerical Results



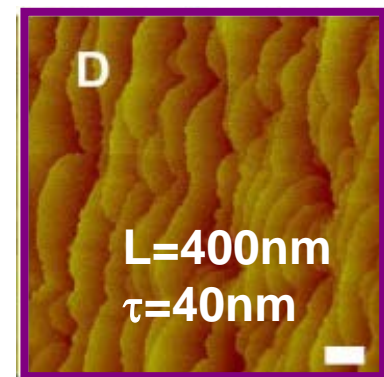
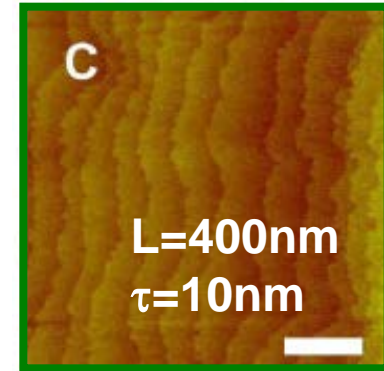
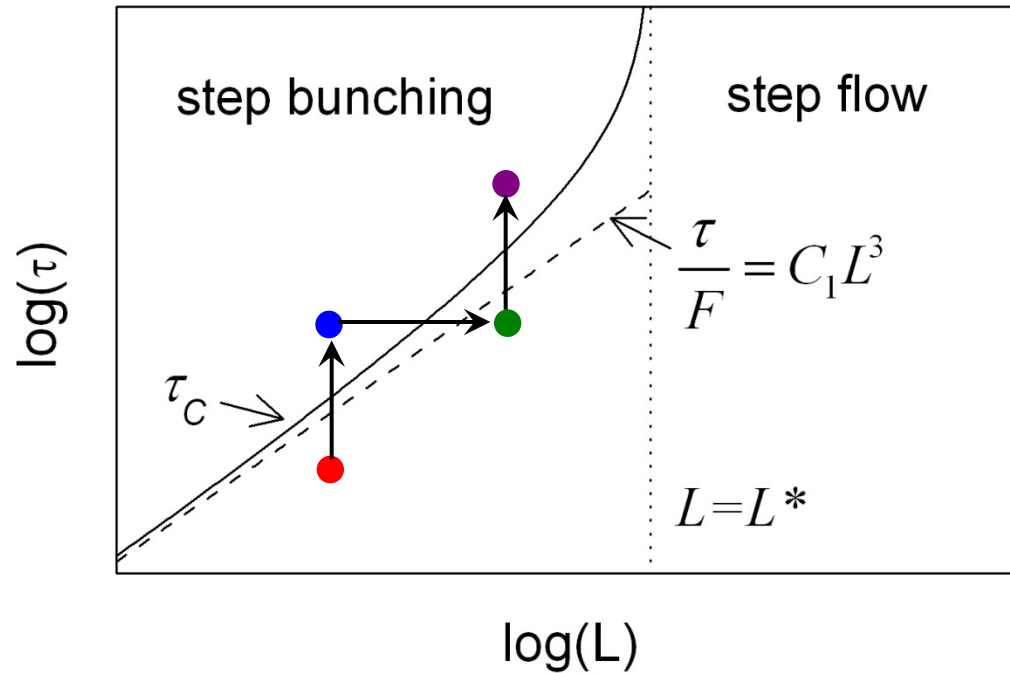
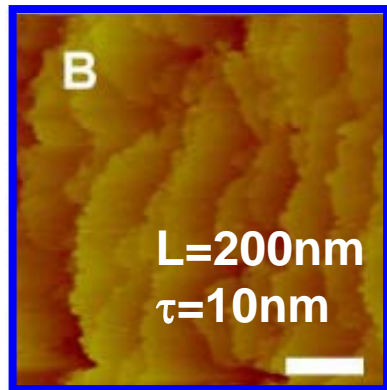
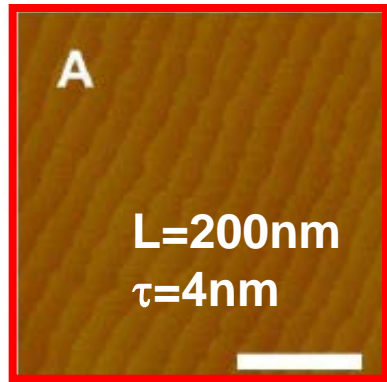
**Scaling behavior of film growth is also numerically confirmed**



$$\frac{\tau_C}{F} = \frac{\tau'_C}{F'}$$

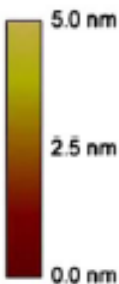


# Experiments & Theory



[M. Yoon et al., Phys. Rev. Lett. 99, 055503 (2007)]

- ❖ Existence of a critical thickness !
- ❖ Under certain thickness, always step flow observed at a given conditions
- ❖ System parameters can be extracted



# Summary on Step Bunching

- ❖ *Persistent* step flow is possible by satisfying the conditions of averting island formation and averting step bunching.
- ❖ Even in the step bunching growth region, step flow can be achieved below critical thickness, where scaling behavior between critical thickness and flux is predicted.
- ❖ System parameters can be extracted using the growth-mode diagram and scaling behavior.